HydroInform®
Water Information Technologies

HydroPad Reference manual

Surface profile calculations

In order to calculate the surface profile the channel is divided into small subsections.

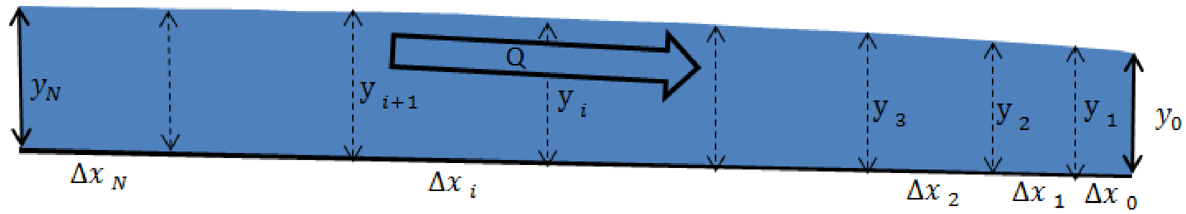

Figure 1: Channel divided into small sub-sections.

Calculations start from the downstream boundary of the channel where the water depth y_0 is known (the boundary condition). Based on the theory and equations presented below the upstream water depth (y_1) for the first subsection is calculated. Subsequent depths are calculated in the same manner, each time using the previous calculated depth as downstream boundary.

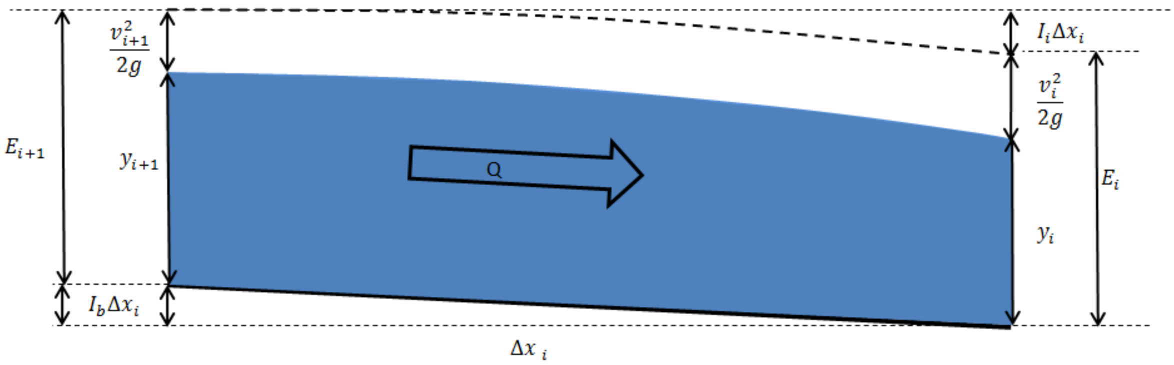

Figure 2: Channel sub-section

For each sub-section the following equations apply.

The conservation of energy equation (the Bernoullis equation) for the subsection:

$$E_{i+1} + I_b \Delta x_i = E_i + I_i \Delta x_i \Leftrightarrow E_i - E_{i+1} + (I_i - I_b)\Delta x_i = 0 \quad (1)$$

Where E_i and E_{i+1} are the specific energy at locations *(i)* and*(i+1)*, respectively. I_b is the slope of the river bed and I_i is the energy gradient. The specific energy can be calculated from equations 2 and 3 below.

$$E_i = \frac{v_i^2}{2g} + y_i \quad (2)$$

$$E_{i+1} = \frac{v_{i+1}^2}{2g} + y_{i+1} \quad (3)$$

Where y_i and y_{i+1} are the water depths a locations (*i)* and (*i+1)*, respectively. V_i and v_{i+1} are the velocities averaged over the cross-section areas, at locations (*i)* and (*i+1)*, respectively. *g* is the gravitational acceleration.

The energy gradient (*I*) can be calculated from the Manning equation:

$$I = \left(\frac{v}{R^{2/3}M}\right)^2 \qquad (4)$$

Where M is the manning number and R is the hydraulic radius.

$$R = \frac{A}{P} \qquad (5)$$

Where A is the cross section of the flow perpendicular to the flow direction and P is the wetted perimeter (the length of the solid channel surface in contact with the water).

HydroPad use trapezoidal cross sections as defined from figure 3 below is used.

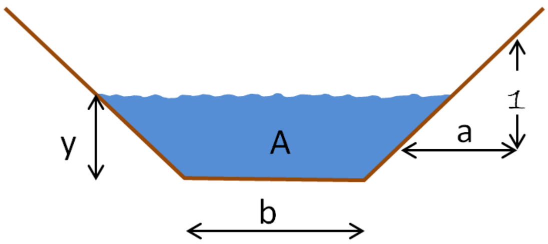

Figure 3: Trapezoidal cross section definition

For trapezoidal cross sections we have:

$$A = y(b + ay) \qquad (6)$$

$$P = b + 2y\sqrt{(1 + a^2)} \qquad (7)$$

Velocities are calculated from:

$$v = \frac{Q}{A} \qquad (8)$$

where *Q* is the constant flow in the channel.

When the depth (y) in known the energy gradient can be calculated from equations 4,5,6,7,8. In order to get the best approximation for the energy loss for the sub-section, the energy gradient in equation 1 is calculated based on the depth $y = (y_i + y_{i+1})/2$.

Now all equations needed have been described, and equation 1 can be solved with respect to y_i . I HydroPad this is done using the numerical iterative bisection method. Note that since we assume the flow to always be within the sub-critical regime equation 1 will always have one and only one root.

In order to improve the computational performance, different lengths of the individual sub-sections (Δx) are used. When the energy gradient is large small values for Δx is used and vice versa.

Normal depth

Fully developed flow through a prismatic channel (a channel with constant slope and cross section, such as the channel in HydroPad) at constant depth y_n is termed flow at normal depth or uniform flow. The normal depth can be calculated as the depth for which the energy gradient equals the bottom slope.

$$I_b = I = \left(\frac{v}{R^{2/3}M}\right)^2 = \left(\frac{Q}{AR^{2/3}M}\right)^2 = \left(\frac{Q}{y_n(b+ay_n)R^{2/3}M}\right)^2 \tag{9}$$

For a given flow rate (Q), Manning number (M), and bottom slope (I_b) the normal depth (y_n) can be found using an iterative solution scheme.

Critical depth

The critical depth is the depth for which the average velocity of the streaming water for a given flow rate equals the wave speed. This means that, seen from the perspective of an observer located on the river bank, waves on top of the water surface will move downstream of the depth is below critical depth and move upstream if the depth is above the critical depth. For depths below the critical depth (wave speed slower than the average speed of the flowing water) the flow is called subcritical. For depth above the critical depth (wave speed faster than the average speed of the flowing water) the flow is called supercritical. For flow regimes with subcritical flow the downstream conditions will influence the flow and for supercritical flow regimes the flow will be influenced by conditions upstream. HydroPad is limited to subcritical conditions only.

The wave speed v_w can be determined from equation 10 below

$$v_w = \sqrt{gy_h} = \sqrt{g\frac{A}{b_s}} = \sqrt{g\frac{y(b+ay_c)}{b+2ay_c}} \tag{10}$$

Where A y_h is the hydraulic depth, y_c is the critical depth, A is the flow area, b_s is the surface width, which can be calculated from equation 11.

$$b_s = b + 2ay_c \tag{11}$$

Where b, a, and y are defined as shown in figure 3.

We have critical depth in the channel when the wave speed equals the speed of the flowing water.

$$v = v_w \tag{12}$$

Now by inserting equations 8, and 11 into equation 12 we get

$$\frac{Q}{y(b+ay_c)} = \sqrt{g\frac{y(b+ay_c)}{b+2ay_c}} \tag{13}$$

Y_c can be calculated from equation 13 by use of an iterative numerical scheme.

Critical slope

The critical slope I_c is the bedslope for which $y_n = y_c$. From equation 9 we get

$$I_c = \left(\frac{Q}{y_c(b+ay_c)R^{2/3}M}\right)^2 \qquad (14)$$